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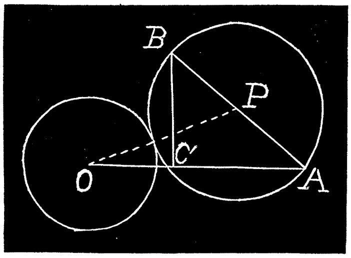
**Solution by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.**

Let $O$ be the center of the given circle, radius$=r$, $A$ the given point, $OA=b$, $OC=x$, $BC=y$. Denoting the radius of the variable circle $P$ by $\rho$, then



$$(r+\rho)^2=\tfrac{1}{2}y^2+\tfrac{1}{4}(b+x)^2 \text{ and } 4\rho^2=y^2+(b-x)^2.$$

Eliminating $\rho$, we obtain the equation of the locus of $B$,

$$y^2=\frac{b^2-r^2}{r^2}(x^2-r^2),$$

a hyperbola, with semi-conjugate axes $r$ and $\sqrt{(b^2-r^2)}$. Consequently, $A$ will be a focus.

Also solved by J. S. Brown.

313. **Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.**

Prove that an algebraic curve of odd degree which is symmetrical with respect to a center has the center on the curve.

I. **Solution by DR. L. E. DICKSON, Associate Professor of Mathematics, The University of Chicago.**

By a transformation of coordinates we may assume that the center is at the origin. If the resulting equation of odd degree is $f(x,\ y)=0$, then must $f(-x,\ -y)=0$ for all values $x$, $y$ satisfying $f(x,\ y)=0$. But the terms of highest degree in $f(x,\ y)$ change sign when $x$ and $y$ are changed in sign. Hence $f(-x,\ -y)\equiv -f(x,\ y)$, so that the terms of even degree vanish. In particular, the constant term is zero, so that the origin lies on the curve.

II. **Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.**

If in the equation of any locus we put $-x$, $-y$ for $x$, $y$, respectively, we get a locus symmetrical to the given locus with respect to the origin. In case the locus is symmetrical to itself, the latter equation will represent the same locus and differ from the original equation only by a constant multiplier. Let

$$0=u_0+u_1+u_2+u_3+\ldots+u_r+\ldots$$

be the equation of a locus symmetrical with respect to the origin, $u_n$ denoting the terms of degree $n$, and $r$ being an *odd* integer. Another equation to this locus, obtained by putting $-x$, $-y$ for $x$, $y$, respectively, and changing the sign of each term, is

$$0=-u_0+u_1-u_2+u_3-\ldots+u_r-\ldots$$

Since each equation contains the term $u_r$, the constant multiplier is unity.

Therefore

$$0=u_0=u_2=u_4=\ldots$$

That is, *if a locus symmetrical with respect to the origin contains one term of odd degree, it contains no absolute term and no terms of even degree.* A further expansion of the problem leads to the theorem that *if a locus symmetrical with respect to the origin contains one term of even degree, all the terms are of even degree.*

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## CALCULUS.

239. **Proposed by L. H. MacDONALD, A. M., Ph. D., Sometime Tutor in the University of Cambridge, Jersey City, N. J.**

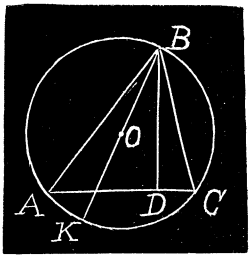
Of all triangles inscribed in a circle, find that which has the greatest perimeter.

**Solution by C. N. SCHMALL, 89 Columbia Street, New York City, and REV. J. H. MEYER, S. J., Augusta, Ga.**

Let $ABC$ be the required triangle; $O$, the center of the given circle; $BK,=2r$, the diameter of the circle; $\angle BAC=\phi$, and $\angle BCA=\psi$. Then the perimeter, $p,=AB+BC+AC=$maximum... (1).

But $AB.BC=2r.BD$, whence $BC=2r\frac{BD}{AB}=2r\sin\phi$.

Also $\frac{AB}{BC}=\frac{\sin\psi}{\sin\phi}$, whence $AB=BC\frac{\sin\psi}{\sin\phi}$, which, from the previous equation, $=2r\sin\psi$. We also have, from the law of signs,



$$AC=\frac{\sin(\phi+\psi)}{\sin\phi}BC=2r\sin(\phi+\psi).$$

Substituting in (1), we have

$$p=2r[\sin\phi+\sin\psi+\sin(\phi+\psi)]\ldots(2).$$

Hence, for a maximum or minimum,

$$\frac{\partial p}{\partial\phi}=\cos\phi+\cos(\phi+\psi)=0,\ \text{and}\ \frac{\partial p}{\partial\psi}=\cos\psi+\cos(\phi+\psi)=0.$$

Hence, $\cos\phi=\cos\psi$ and $\phi=\psi$. Since $\cos\phi+\cos(\phi+\psi)=0$, we have, by substitution, $\cos\phi+\cos2\phi=0$, or $\cos\phi+2\cos^2\phi-1=0$, whence, $\cos\phi=\frac{1}{2}$, and therefore $\phi=60^\circ=\psi$.

Hence, the triangle is equilateral.

It is easy to show that $\frac{\partial^2p}{\partial\phi^2}.\frac{\partial^2p}{\partial\psi}>\frac{\partial^2p}{\partial\phi\,\partial\psi}$, and that, therefore, the triangle is a maximum.